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A long circular cylinder with a circumferential edge crack subjected to a uniform shearing stress

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Abstract

This paper contains an analysis of the stress distribution in a long circular cylinder of isotropic elastic material with a circumferential edge crack when it is deformed by the application of a uniform shearing stress. The crack with its center on the axis of the cylinder lies on the plane perpendicular to that axis, and the cylindrical surface is stress-free. By making a suitable representation of the stress function for the problem, the problem is reduced to the solution of a pair of singular integral equations. This pair of singular integral equations is solved numerically, and the stress intensity factor due to the effect of the crack size is tabulated. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: Circumferential edge crack; Cylinder; Shearing stress

1. Introduction

Problem of determining the distribution of stress in a long circular cylinder containing a penny-shaped crack has been investigated by relatively many writers. We can list works by Collins (1962), Sneddon and Tait (1963), Sneddon and Welch (1963), Tchuchida and Uchiyama (1980), Dhaliwal et al. (1979), Ban and Zhang (1992), Loboda and Sheveleva (1995), and Wang (1996).

However, solutions to the problem concerning an edge circumferential crack in a long circular cylinder are relatively few. Keer et al. (1977) considered an infinitely long isotropic circular cylinder with a circumferential edge crack subjected to tension, whereas Atsumi and Shindo (1979) considered the same problem when the cylinder is of transversely isotropic material. All of these investigations concerned with axisymmetric problems, however, little attention seems to be given to the asymmetric analysis concerning a long circular cylinder. The recent analyses by Lee (1997, 1999) investigated the asymmetric problems involving a long cylinder.

In this paper we derive the solution of the problem determining the distribution of stress in a long circular cylinder of elastic material (Fig. 1), whose surface is stress-free, when it is deformed by the application of the shearing stress at the end of the cylinder which contains a circumferential edge crack

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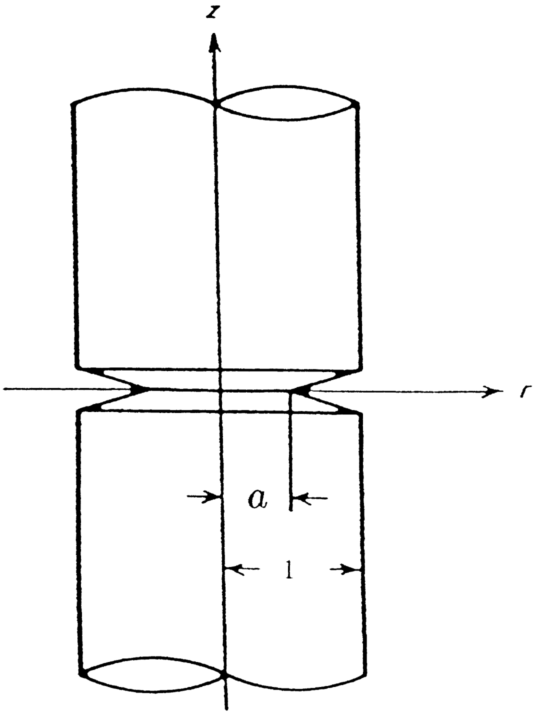

Fig. 1. A long circular cylinder with circumferential edge crack subjected uniform shearing stress.

situated with its center on the axis of the cylinder and its plane perpendicular to that axis. An analogous problem concerning a penny-shaped crack has been considered by the present author (Lee, 2001).

Radial shear problem has been considered in Kassir and Sih (1975) for the infinite elastic medium and by Danyluk et al. (1991) for the cylinder of finite radius of transversely isotropic medium. Such radial shear would, in real life situation, be difficult to realize, whereas the uniform shear problem such as the present one can be examined experimentally fairly easily.

In Section 2 general equations and formulation of the problem are given, and in Sections 3 and 4 by the use of the stress and displacement fields, and employing Fourier transform, a pair of singular integral equations is derived. And finally, a numerical example is given and quantities of physical interest are obtained.

2. General equation

Consider a circular cylinder of the radius c having a circumferential edge crack whose inner radius is equal to a. We take the center of the crack as the origin of the cylindrical coordinates, (r, θ, z) and the axis of the cylinder to be z-axis. The central plane of the crack is taken to lie on the plane $z = 0$. Suppose that the cylinder is subjected to a constant shearing stress at the end. By introducing an appropriate function, we can convert the problem to a mixed boundary value problem where the crack is subjected to a uniform shearing stress while all stresses at the ends vanish. For instance, taking $\psi = Crz$ in (2.10) in the subsequent page, we see that this function produces no stresses except uniform shear stresses. Adding this function to the potential function leads us to the goal.

It is considered on the surface $z = 0$, the crack is subjected to a uniform shearing stress S in the $\theta = \pi$-direction. Interior to the circle the surface displacements u, v are zero; the surface $z = 0$ is assumed to be free from normal tractions. Then the problem determining the distribution of stress in the vicinity of the crack is equivalent to that of finding the distribution of stress in the semi-infinite cylinder $z \geqslant 0$, $0 \leqslant r \leqslant 1$. The boundary conditions for the present problem can be mathematically stated as follows:

On $z = 0$:
$$\tau_{rz} = S\cos\theta, \quad a < r \leqslant 1, \tag{2.1}$$
$$\tau_{\theta z} = -S\sin\theta, \quad a < r \leqslant 1, \tag{2.2}$$
$$\sigma_z = 0, \quad 0 \leqslant r \leqslant 1, \tag{2.3}$$
$$u = 0, \quad 0 \leqslant r < a, \tag{2.4}$$
$$v = 0, \quad 0 \leqslant r < a, \tag{2.5}$$
On $r = 1$:
$$\tau_{rz} = 0, \tag{2.6}$$
$$\tau_{r\theta} = 0, \tag{2.7}$$
$$\sigma_r = 0. \tag{2.8}$$

If we introduce biharmonic and harmonic functions Φ, and ψ, so that

$$\nabla^4\Phi = \nabla^2\psi = 0,$$

then the needed components of the displacement vector in terms of them are expressed by the equation from Muki (1960, p. 403)

$$\begin{aligned} u &= -\frac{\partial^2\Phi}{\partial r\partial z} + \frac{2}{r}\frac{\partial\psi}{\partial\theta}, \\ v &= -\frac{\partial^2\Phi}{r\partial\theta\partial z} - 2\frac{\partial\psi}{\partial r}, \end{aligned} \tag{2.9}$$

and the stress field is given by the equations,

$$\begin{aligned} \frac{\sigma_r}{2\mu} &= \frac{\partial}{\partial z}\left(v\nabla^2\Phi - \frac{\partial^2\Phi}{\partial r^2}\right) + \frac{2}{r}\frac{\partial^2\psi}{\partial\theta\partial r} - \frac{2}{r^2}\frac{\partial\psi}{\partial\theta}, \\ \frac{\sigma_\theta}{2\mu} &= \frac{\partial}{\partial z}\left(v\nabla^2\Phi - \frac{1}{r}\frac{\partial\Phi}{\partial r} - \frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}\right) - \frac{2}{r}\frac{\partial^2\psi}{\partial\theta\partial r} + \frac{2}{r^2}\frac{\partial\psi}{\partial\theta}, \\ \frac{\sigma_z}{2\mu} &= \frac{\partial}{\partial z}\left\{(2-v)\nabla^2\Phi - \frac{\partial^2\Phi}{\partial z^2}\right\}, \\ \frac{\tau_{rz}}{2\mu} &= \frac{\partial}{\partial r}\left\{(1-v)\nabla^2\Phi - \frac{\partial^2\Phi}{\partial z^2}\right\} + \frac{\partial^2\psi}{r\partial\theta\partial z}, \\ \frac{\tau_{\theta z}}{2\mu} &= \frac{\partial}{r\partial\theta}\left\{(1-v)\nabla^2\Phi - \frac{\partial^2\Phi}{\partial z^2}\right\} - \frac{\partial^2\psi}{\partial r\partial z}, \\ \frac{\tau_{r\theta}}{2\mu} &= \frac{\partial^2}{r\partial\theta\partial z}\left(\frac{\Phi}{r} - \frac{\partial\Phi}{\partial r}\right) - 2\frac{\partial^2\psi}{\partial r^2} - \frac{\partial^2\psi}{\partial z^2}. \end{aligned} \tag{2.10}$$

where μ is the modulus of rigidity, and v is Poisson's ratio. In addition to (2.9) and (2.10), for the solution of the present problem we need more expressions for the displacements and stresses. This can be obtained from Lee (2001).

The displacement vector is found as

$$u = -2(1-v)\frac{\partial(f+g)}{\partial z}\cos\theta + z\frac{\partial F}{\partial r},$$
$$v = -2(1-v)\frac{\partial(g-f)}{\partial z}\sin\theta + z\frac{1}{r}\frac{\partial F}{\partial \theta}, \tag{2.11}$$
$$w = -(1-2v)F + z\frac{\partial F}{\partial r},$$

where $f(r,z)$ satisfies

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{\partial^2 f}{\partial z^2} = 0 \tag{2.12}$$

and $g(r,z)$ satisfies

$$\frac{\partial^2 g}{\partial r^2} + \frac{1}{r}\frac{\partial g}{\partial r} - \frac{4}{r^2}g + \frac{\partial^2 g}{\partial z^2} = 0 \tag{2.13}$$

and

$$F = \left(\frac{\partial f}{\partial r} + \frac{\partial g}{\partial r} + \frac{2g}{r}\right)\cos\theta, \qquad \nabla^2 F = 0.$$

The components of stress tensor are

$$\frac{\sigma_r}{2\mu} = -2(1-v)\frac{\partial^2(f+g)}{\partial z\partial r}\cos\theta - 2v\frac{\partial F}{\partial z} + z\frac{\partial^2 F}{\partial r^2},$$
$$\frac{\sigma_z}{2\mu} = z\frac{\partial^2 F}{\partial z^2},$$
$$\frac{\tau_{r\theta}}{2\mu} = -(1-v)\frac{\partial}{\partial z}\left(-\frac{\partial f}{\partial r} + \frac{\partial g}{\partial r} - \frac{2g}{r}\right)\sin\theta + z\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial F}{\partial \theta}\right), \tag{2.14}$$
$$\frac{\tau_{rz}}{2\mu} = -(1-v)\frac{\partial^2(f+g)}{\partial z^2}\cos\theta + v\frac{\partial F}{\partial r} + z\frac{\partial^2 F}{\partial r\partial z},$$
$$\frac{\tau_{\theta z}}{2\mu} = -(1-v)\frac{\partial^2(g-f)}{\partial z^2}\sin\theta + v\frac{1}{r}\frac{\partial F}{\partial \theta} + z\frac{1}{r}\frac{\partial^2 F}{\partial \theta\partial z}.$$

3. Derivation of integral equations

In this section we derive a pair of singular integral equations. For this purpose we use the displacement vector obtained by adding (2.9) and (2.11). For the stress field we add (2.10) and (2.14). Suitable functions for f, g, Φ, and ψ are chosen to be

$$f(r,z) = \frac{1}{2\mu}\int_0^\infty \xi^{-1}\alpha(\xi)J_0(\xi r)e^{-\xi z}\,d\xi, \tag{3.1}$$

$$g(r,z) = \frac{1}{2\mu}\int_0^\infty \xi^{-1}\beta(\xi)J_2(\xi r)e^{-\xi z}\,d\xi, \tag{3.2}$$

$$\Phi(r,\theta,z) = \frac{1}{2\mu}\int_0^\infty \{A(\xi)rI_2(\xi r) + B(\xi)I_1(\xi r)\}\cos\xi z\,d\xi\cos\theta, \tag{3.3}$$

$$\psi(r,\theta,z) = \frac{1}{2\mu}\int_0^\infty C(\xi)I_1(\xi r)\sin\xi z\,\mathrm{d}\xi\sin\theta, \tag{3.4}$$

where J_0, J_2 are the Bessel function of the first kind and I_1, I_2 are the modified Bessel functions of the first kind. With these choice of functions, we see that condition (2.3) is automatically satisfied.

On $z = 0$, we have

$$\frac{u}{\cos\theta} = \frac{1-v}{\mu}\int_0^\infty \{\alpha(\xi)J_0(\xi r) + \beta(\xi)J_2(\xi r)\}\,\mathrm{d}\xi, \tag{3.5}$$

and, we also have

$$\frac{v}{\sin\theta} = \frac{1-v}{\mu}\int_0^\infty \{-\alpha(\xi)J_0(\xi r) + \beta(\xi)J_2(\xi r)\}\,\mathrm{d}\xi. \tag{3.6}$$

If one defines

$$-\frac{\mu}{2(1-v)}\frac{\partial}{\partial r}\left(\frac{u}{\cos\theta} - \frac{v}{\sin\theta}\right) = \begin{cases} r^{-1}g(r), & a < r \leqslant 1 \\ 0, & 0 \leqslant r \leqslant a \end{cases} \tag{3.7}$$

then $\alpha(\xi)$ is determined as

$$\alpha(\xi) = \int_a^1 g(t)J_1(\xi t)\,\mathrm{d}t, \tag{3.8}$$

Similarly, if one defines

$$\frac{\mu}{2(1-v)}\frac{\partial}{\partial r}\left\{r^2\left(\frac{u}{\cos\theta} + \frac{v}{\sin\theta}\right)\right\} = \begin{cases} rh(r), & a < r \leqslant 1 \\ 0, & 0 \leqslant r \leqslant a \end{cases} \tag{3.9}$$

then $\beta(\xi)$ is determined as

$$\beta(\xi) = \int_a^1 h(t)J_1(\xi t)\,\mathrm{d}t, \tag{3.10}$$

Now τ_{rz} on $z = 0$ is

$$\begin{aligned}\frac{\tau_{rz}}{\cos\theta} = &-\int_0^\infty \xi\{\alpha(\xi)J_0(\xi r) + \beta(\xi)J_2(\xi r)\}\mathrm{d}\xi + \frac{v}{r}\int_0^\infty \{\alpha(\xi) + \beta(\xi)\}J_1(\xi r)\mathrm{d}\xi \\ &+ \int_0^\infty \xi^2\left[A(\xi)\left\{\left(\frac{2(1-v)}{\xi r} + \xi r\right)I_1(\xi r) + (1-2v)I_2(\xi r)\right\}\right. \\ &\left. + B(\xi)\xi\left\{I_2(\xi r) + \frac{1}{\xi r}I_1(\xi r)\right\} + C(\xi)\frac{1}{\xi r}I_1(\xi r)\right]\mathrm{d}\xi = S, \quad a < r \leqslant 1.\end{aligned} \tag{3.11}$$

If we use (3.8) and (3.10) in (3.11), we obtain following singular integral equation for determining $g(t)$ and $h(t)$:

$$\frac{2}{\pi}\int_a^1\{g(t)-h(t)\}R(r,t)\,\mathrm{d}t+\frac{v}{r}\int_a^1 g(t)H_{11}(r,t)\,\mathrm{d}t+\frac{(v-2)}{r}\int_a^1 h(t)H_{11}(r,t)\,\mathrm{d}t$$
$$+\int_0^\infty \xi^2\Big[A(\xi)\Big\{\Big(\frac{2(1-v)}{\xi r}+\xi r\Big)I_1(\xi r)+(1-2v)I_2(\xi r)\Big\}$$
$$+B(\xi)\xi\Big\{I_2(\xi r)+\frac{1}{\xi r}I_1(\xi r)\Big\}+C(\xi)\frac{1}{\xi r}I_1(\xi r)\Big]\,\mathrm{d}\xi=S,\quad a<r\leqslant 1, \tag{3.12}$$

where

$$R(r,t)=\begin{cases}\frac{1}{r^2-t^2}E(\frac{r}{t}), & r<t,\\ \frac{r}{t}\frac{1}{r^2-t^2}E(\frac{t}{r})-\frac{1}{rt}K(\frac{t}{r}), & r>t.\end{cases} \tag{3.13}$$

(Here K and E are complete elliptic integrals of the first and second kind, respectively) and $H_{11}(r,t)$ is defined by

$$H_{11}(r,t)=\int_0^\infty J_1(\xi r)J_1(\xi t)\mathrm{d}\xi$$
$$=\begin{cases}\frac{t}{2r^2}\,{}_2F_1\Big(\frac{3}{2},\frac{1}{2};2;\frac{t^2}{r^2}\Big)=\frac{2}{\pi t}\big[K\big(\frac{t}{r}\big)-E\big(\frac{t}{r}\big)\big], & t<r,\\ \frac{r}{2t^2}\,{}_2F_1\Big(\frac{3}{2},\frac{1}{2};2;\frac{r^2}{t^2}\Big)=\frac{2}{\pi r}\big[K\big(\frac{r}{t}\big)-E\big(\frac{r}{t}\big)\big], & r<t.\end{cases} \tag{3.14}$$

Similarly from (2.2), we have now $\tau_{\theta z}$ on $z=0$ as follows:

$$\frac{\tau_{\theta z}}{\sin\theta}=-(1-v)\int_0^\infty \xi\{-\alpha(\xi)J_0(\xi r)+\beta(\xi)J_2(\xi r)\}\mathrm{d}\xi-\frac{v}{r}\int_0^\infty\{-\alpha(\xi)+\beta(\xi)\}J_1(\xi r)\,\mathrm{d}\xi$$
$$-\frac{1}{r}\int_0^\infty \xi[A(\xi)\{2(1-v)I_1(\xi r)+\xi rI_2(\xi r)\}+B(\xi)\xi I_1(\xi r)+C(\xi)\{\xi rI_2(\xi r)+I_1(\xi r)\}]\,\mathrm{d}\xi=-S,\quad a<r\leqslant 1. \tag{3.15}$$

If we substitute (3.8) and (3.10) for α and β into (3.15) we obtain another singular integral equation for determining $g(t)$ and $h(t)$:

$$-(1-v)\frac{2}{\pi}\int_a^1\{g(t)+h(t)\}R(r,t)\mathrm{d}t+\frac{v}{r}\int_a^1 g(t)H_{11}(r,t)\,\mathrm{d}t+\frac{(v-2)}{r}\int_a^1 h(t)H_{11}(r,t)$$
$$-\frac{1}{r}\int_0^\infty \xi[A(\xi)\{2(1-v)I_1(\xi r)+\xi rI_2(\xi r)\}+B(\xi)\xi I_1(\xi r)+C(\xi)\{\xi rI_2(\xi r)+I_1(\xi r)\}]\,\mathrm{d}\xi=-S,\quad a<r\leqslant 1. \tag{3.16}$$

Substituting (3.8) and (3.10) into (3.5) and (3.6) and using condition (2.4) and (2.5), the following additional condition is obtained.

$$\int_a^1 t^{-1}g(t)\,\mathrm{d}t=0. \tag{3.17}$$

4. Conditions on the surface of the cylinder

We now complete the solution by satisfying the boundary conditions on the surface of the cylinder. It is easily shown that the value of τ_{rz} on the surface $r=1$ is given by the equation

$$\frac{\tau_{rz}(1,\theta,z)}{\cos\theta} = \int_0^\infty \xi^2[\{2(1-v)I_1'(\xi)+\xi I_1(\xi)-I_2(\xi)\}A(\xi)+\xi I_1'(\xi)B(\xi)+\frac{1}{\xi}I_1(\xi)C(\xi)]\cos\xi z\,\mathrm{d}\xi$$
$$+\int_0^\infty[\alpha(\eta)\{-\eta J_0(\eta)+vJ_1(\eta)+z\eta^2J_1'(\eta)\}+\beta(\eta)\{-\eta J_2(\eta)+vJ_1(\eta)-z\eta^2J_1'(\eta)\}]\mathrm{e}^{-\eta z}\,\mathrm{d}\eta. \tag{4.1}$$

Boundary condition (2.6) can be written in an alternative form as

$$\mathscr{F}_c[\tau_{rz}(1,\theta,z);z\to\xi]=0. \tag{4.2}$$

From which, we obtain

$$\{2(1-v)I_1'(\xi)+\xi I_1(\xi)-I_2(\xi)\}A(\xi)+\xi I_1'(\xi)B(\xi)+\frac{1}{\xi}I_1(\xi)C(\xi)$$
$$=-\frac{2}{\pi}\int_0^\infty\left[\alpha(\eta)\left\{\frac{(v-1)}{\xi^2}\frac{\eta J_1(\eta)}{\xi^2+\eta^2}-2\frac{\eta^2J_0(\eta)}{(\xi^2+\eta^2)^2}+2\frac{\eta J_1(\eta)}{(\xi^2+\eta^2)^2}\right\}\right.$$
$$\left.+\beta(\eta)\left\{-2\frac{\eta^2J_2(\eta)}{(\xi^2+\eta^2)^2}+\frac{(v-1)}{\xi^2}\frac{\eta J_1(\eta)}{\xi^2+\eta^2}+2\frac{\eta J_1(\eta)}{(\xi^2+\eta^2)^2}\right\}\right]\mathrm{d}\eta. \tag{4.3}$$

Therefore if we substitute $\alpha(\eta)$ and $\beta(\eta)$ from (3.8) and (3.10) into (4.3) and make use of formulae in Appendix which are obtained from Erdélyi et al. (1954), we obtain one equation connecting $A(\xi)$, $B(\xi)$, $C(\xi)$ and $g(t)$, $h(t)$,

$$\{(2(1-v)+\xi^2)I_1(\xi)+(1-2v)\xi I_2(\xi)\}A(\xi)+\xi\{I_1(\xi)+\xi I_2(\xi)\}B(\xi)+I_1(\xi)C(\xi)$$
$$=-\{(\xi^2+v+1)K_1(\xi)-\xi K_2(\xi)\}i(\xi)+\{-K_1(\xi)+\xi K_2(\xi)\}j(\xi)+\{(3-v+\xi^2)K_1(\xi)-\xi K_2(\xi)\}k(\xi), \tag{4.4}$$

where

$$i(\xi)=\frac{2}{\pi\xi}\int_a^1 g(t)I_1(\xi t)\,\mathrm{d}t,$$
$$j(\xi)=\frac{2}{\pi\xi}\int_a^1\{g(t)-h(t)\}\xi tI_2(\xi t)\,\mathrm{d}t, \tag{4.5}$$
$$k(\xi)=\frac{2}{\pi\xi}\int_a^1 h(t)I_1(\xi t)\,\mathrm{d}t.$$

Next, $\tau_{r\theta}$ on the surface $r=1$ is given by the equation

$$\frac{\tau_{r\theta}(1,\theta,z)}{\sin\theta}=\int_0^\infty\xi^2[-I_2'(\xi)A(\xi)-I_2(\xi)B(\xi)+\left\{-I_1(\xi)+\frac{2}{\xi}I_2(\xi)\right\}C(\xi)]\sin\xi z\,\mathrm{d}\xi$$
$$+\int_0^\infty[\alpha(\eta)\{(1-v)\eta J_1(\eta)-z\eta J_2(\eta)\}+\beta(\eta)\{(1-v)[\eta J_1(\eta)-4J_2(\eta)]+z\eta J_2(\eta)\}]\mathrm{e}^{-\eta z}\,\mathrm{d}\eta. \tag{4.6}$$

The boundary condition (2.7) can be written as

$$\mathscr{F}_s[\tau_{r\theta}(1,\theta,z);z\to\xi]=0. \tag{4.7}$$

From which we obtain

$$\left\{-I_1(\xi)+\frac{2}{\xi}I_2(\xi)\right\}A(\xi)-I_2(\xi)B(\xi)+\left\{-I_1(\xi)+\frac{2}{\xi}I_2(\xi)\right\}C(\xi)$$
$$=-\frac{2}{\pi}\int_0^\infty\left[\alpha(\eta)\left\{\frac{(1-v)}{\xi}\frac{\eta J_1(\eta)}{\xi^2+\eta^2}-\frac{2}{\xi}\frac{\eta^2J_2(\eta)}{(\xi^2+\eta^2)^2}\right\}+\beta(\eta)\left\{[\eta J_1(\eta)-4J_2(\eta)]\frac{1-v}{\xi(\xi^2+\eta^2)}+\frac{2}{\xi}\frac{\eta^2J_2(\eta)}{(\xi^2+\eta^2)^2}\right\}\right]\mathrm{d}\eta \tag{4.8}$$

If we substitute from (3.8) and (3.10) into (4.8) and make use of equations in the Appendix, we obtain

$$\{-\xi I_1(\xi)+2I_2(\xi)\}A(\xi)-\xi I_2(\xi)B(\xi)+\{-\xi I_1(\xi)+2I_2(\xi)\}C(\xi)$$
$$=v\xi K_1(\xi)i(\xi)-K_2(\xi)j(\xi)-\{(2-v)\xi K_1(\xi)+4(1-v)K_2(\xi)\}k(\xi). \tag{4.9}$$

Finally, σ_r on the surface $r=1$ is given by the equation

$$\frac{\sigma_r}{\cos\theta}=-\int_0^\infty\xi^2\left[\left\{(2v-1)I_1(\xi)-\left(\xi+\frac{2}{\xi}\right)I_2(\xi)\right\}A(\xi)-\xi I_1''(\xi)B(\xi)-\frac{2}{\xi}I_2(\xi)C(\xi)\right]\sin\xi z\,\mathrm{d}\xi$$
$$+\int_0^\infty[\alpha(\eta)\{-2\eta J_1(\eta)-z[-\eta^2J_1(\eta)+\eta J_2(\eta)]\}$$
$$+\beta(\eta)\{2\eta J_1(\eta)-4(1-v)J_2(\eta)+z[-\eta^2J_1(\eta)+\eta J_2(\eta)]\}]\mathrm{e}^{-\eta z}\,\mathrm{d}\eta. \tag{4.10}$$

Boundary condition (2.7) can be written in an alternative form as

$$\mathscr{F}_s[\sigma_r(1,\theta,z);z\to\xi]=0. \tag{4.11}$$

From which we obtain

$$\left\{(2v-1)I_1(\xi)-\left(\xi+\frac{2}{\xi}\right)I_2(\xi)\right\}A(\xi)-\xi I_1''(\xi)B(\xi)-\frac{2}{\xi}I_2(\xi)C(\xi)$$
$$=\frac{2}{\pi}\int_0^\infty\left[\alpha(\eta)\left\{-2\xi\frac{\eta J_1(\eta)}{(\xi^2+\eta^2)^2}-\frac{2}{\xi}\frac{\eta^2J_2(\eta)}{(\xi^2+\eta^2)^2}\right\}+\beta(\eta)\left\{2\xi\frac{\eta J_1(\eta)}{(\xi^2+\eta^2)^2}-\frac{4(1-v)}{\xi}\frac{J_2(\eta)}{\xi^2+\eta^2}+\frac{2}{\xi}\frac{\eta^2J_2(\eta)}{(\xi^2+\eta^2)^2}\right\}\right]\mathrm{d}\eta. \tag{4.12}$$

If we substitute from (3.8) and (3.10) into (4.12) and make use of equations in the Appendix and their derivatives, we obtain

$$\{(2v-1)\xi I_1(\xi)-(\xi^2+2)I_2(\xi)\}A(\xi)-\xi\{\xi I_1(\xi)-I_2(\xi)\}B(\xi)-2I_2(\xi)C(\xi)$$
$$=\xi\{K_1(\xi)-\xi K_2(\xi)\}i(\xi)+\{\xi K_1(\xi)+K_2(\xi)\}j(\xi)-\{\xi K_1(\xi)-(4(1-v)+\xi^2)K_2(\xi)\}k(\xi). \tag{4.13}$$

Now, if we solve (4.4), (4.9) and (4.13) simultaneously for unknowns $A(\xi)$, $B(\xi)$, and $C(\xi)$, we find that

$$A(\xi)=\left[-\frac{K_1}{I_1}+I_2\xi\frac{I_1\xi-2I_2}{\Delta(\xi)}\right]i(\xi)-\frac{I_1(\xi I_1-2I_2)}{\Delta(\xi)}j(\xi)+\left[\frac{K_1}{I_1}-\frac{1}{\Delta(\xi)}\{I_1I_2(\xi^2+4(1-v))-2\xi I_2^2\}\right]k(\xi), \tag{4.14}$$

where $I_1=I_1(\xi)$, $I_2=I_2(\xi)$, $K_1=K_1(\xi)$ and

$$\Delta(\xi)=\xi I_1[4\xi I_1^2I_2+I_1I_2^2\{\xi^2-2(1+v)\}-2\xi I_2^3-\xi^2I_1^3]$$

and

$$\xi B(\xi) = \left[-\frac{K_1}{I_1}2v + \frac{\xi}{\Delta(\xi)}\{ -I_1^2\xi^2 + I_1I_2 2(1+v)\xi - 4vI_2^2\} \right] i(\xi) + \left[-\frac{K_1}{I_1} + \frac{I_1^2 2(1-v)\xi + I_1I_2(2v-2+\xi^2) - 2\xi I_2^2}{\Delta(\xi)} \right] j(\xi) + \left[2v\frac{K_1}{I_1} + \frac{1}{\Delta(\xi)}\{\xi I_1^2[4(1-v)+\xi^2] - I_1I_2\{2(1+v)\xi^2 + 8v(1-v)\} + 4v\xi I_2^2\} \right] k(\xi), \tag{4.15}$$

and

$$C(\xi) = (1-v)\left[\left\{ \frac{K_1}{I_1} + \frac{2\xi I_2^2}{\Delta(\xi)} \right\} i(\xi) - \frac{2I_1I_2}{\Delta(\xi)} j(\xi) + \left\{ \frac{K_1}{I_1} + \frac{1}{\Delta(\xi)}(8I_1I_2 - 4\xi I_1^2 + 2\xi I_2^2) \right\} k(\xi) \right], \tag{4.16}$$

In obtaining (4.14)–(4.16), we used the equation

$$K_2(\xi)I_1(\xi) + K_1(\xi)I_2(\xi) = \frac{1}{\xi}.$$

Therefore, if we substitute $A(\xi)$, $B(\xi)$ and $C(\xi)$ from (4.14)–(4.16) into (3.12), we finally obtain following singular integral equation

$$\frac{2}{\pi}\int_a^1 \{g(t) - h(t)\}R(r,t)\,\mathrm{d}t + \int_a^1 g(t)\left\{ \frac{v}{r}H_{11}(r,t) + K_1(r,t) \right\}\mathrm{d}t + \int_a^1 h(t)\left\{ \frac{v-2}{r}H_{11}(r,t) + K_2(r,t) \right\}\mathrm{d}t = S, \quad a < r \leqslant 1. \tag{4.17}$$

Similarly from (3.16), we obtain another singular integral equation for determining $g(t)$ and $h(t)$:

$$-(1-v)\frac{2}{\pi}\int_a^1 \{g(t) + h(t)\}R(r,t)\,\mathrm{d}t + \int_a^1 g(t)\left\{ \frac{v}{r}H_{11}(r,t) - K_3(r,t) \right\}\mathrm{d}t + \int_a^1 h(t)\left[\frac{(v-2)}{r}H_{11}(r,t) - K_4(r,t) \right]\mathrm{d}t = -S, \quad a < r \leqslant 1. \tag{4.18}$$

The detailed expressions of kernels $K_i(r,t)(i = 1,\dots,4)$ are listed in the Appendix.

The problem is now reduced to the solution of (4.17) and (4.18) under the additional condition (3.17) which is a trivial condition for the result to be determined because it merely refers to the rigid displacement of u and v.

Quantities which are of interest in fracture mechanics are the stress intensity factors defined by the relations

$$k_2 = \lim_{r\to a^-} \sqrt{2(a-r)}\,\tau_{rz}(r,\theta,0), \tag{4.19}$$

$$k_3 = \lim_{r\to a^-} \sqrt{2(a-r)}\,\tau_{\theta z}(r,\theta,0). \tag{4.20}$$

We let

$$r = \frac{d}{2}(s-1) + 1, \quad t = \frac{d}{2}(\tau - 1) + 1, \tag{4.21}$$

where $d = 1 - a$, and in order to facilitate numerical analysis, assume $g(t)$ and $h(t)$ to have the following form:

$$g(t) = S\sqrt{\frac{1-t}{t-a}}G(t), \qquad h(t) = S\sqrt{\frac{1-t}{t-a}}H(t). \tag{4.22}$$

With the aid of (4.21), $g(\tau)$ and $h(\tau)$ can be rewritten as

$$g(\tau) = SG(\tau)\left(\frac{1-\tau}{1+\tau}\right)^{\frac{1}{2}}, \qquad h(\tau) = SH(\tau)\left(\frac{1-\tau}{1+\tau}\right)^{\frac{1}{2}}. \tag{4.23}$$

The stress intensity factors k_2 and k_3 can therefore be expressed in terms of $G(t)$ and $H(t)$ as

$$k_2 = \sqrt{2d}\{G(a) - H(a)\}S\cos\theta/a, \tag{4.24}$$

$$k_3 = -\sqrt{2d}(1-v)\{G(a) + H(a)\}S\sin\theta/a. \tag{4.25}$$

Crack opening displacements, given by integration of (3.7) and (3.9) are

$$\frac{\mu}{1-v}\frac{u(r,\theta,0)}{\cos\theta} = -\int_a^r t^{-1}g(t)\,\mathrm{d}t + \frac{1}{r^2}\int_a^r th(t)\mathrm{d}t, \quad a < r < 1,$$

$$\frac{\mu}{1-v}\frac{v(r,\theta,0)}{\sin\theta} = \int_a^r t^{-1}g(t)\,\mathrm{d}t + \frac{1}{r^2}\int_a^r th(t)\mathrm{d}t, \quad a < r < 1.$$

5. Numerical analysis

In order to obtain numerical solutions of (4.17) and (4.18), substitutions are made by the application of (4.21) and (4.23) to obtain equations of following forms

$$\frac{d}{\pi}\int_{-1}^{1}\left(\frac{1-\tau}{1+\tau}\right)^{\frac{1}{2}}[G(\tau)P_i(s,\tau) + H(\tau)Q_i(s,\tau)]\mathrm{d}\tau = 1, \quad -1 < s < 1 \quad (i = 1,2). \tag{5.1}$$

The numerical solution technique is based on the collocation scheme for the solution of singular integral equations given by Erdogan et al. (1973). This amounts to applying a Gaussian quadrature formula for approximating the integral of a function $f(\tau)$ with weight function $[(1-\tau)/(1+\tau)]^{1/2}$ on the interval $[-1,1]$. Thus, letting n be the number of quadrature points,

$$\int_{-1}^{1}\left(\frac{1-\tau}{1+\tau}\right)^{\frac{1}{2}}f(\tau)\,\mathrm{d}\tau \doteq \frac{2\pi}{2n+1}\sum_{k=1}^{n}(1-\tau_k)f(\tau_k), \tag{5.2}$$

where

$$\tau_k = \cos\left(\frac{2k\pi}{2n+1}\right), \quad k = 1,\ldots,n. \tag{5.3}$$

The solution of the integral equation is obtained by choosing the collocation points:

$$s_i = \cos\left(\frac{2i-1}{2n+1}\right)\pi, \quad i = 1,\ldots,n \tag{5.4}$$

and solving the matrix system for $G^*(\tau_k)$ and $H^*(\tau_k)$:

$$\sum_{k=1}^{n}[G^*(\tau_k)P_i(s_j,\tau_k) + H^*(\tau_k)Q_i(s_j,\tau_k)] = \frac{2n+1}{2d}, \quad j = 1,\ldots,n \ (i = 1,2), \tag{5.5}$$

where

$$G^*(\tau_k) = G(\tau_k)(1-\tau_k), \qquad H^*(\tau_k) = H(\tau_k)(1-\tau_k). \tag{5.6}$$

It can be shown that as both r, t approach 1, the kernels $K_i(r,t)$, $(i = 1,4)$ in (4.17) and (4.18) become unbounded and hence influence the singular nature of the solution. Thus, if $K_i(r,t)$ in Appendix B are, for the brevity of notation, written as

$$K_i(r,t) = \frac{2}{\pi}\int_0^\infty k_i(\xi,r,t)\mathrm{d}\xi \quad (i = 1,\ldots,4),$$

the unbounded terms in $K_i(r,t)$ will be the consequence of the asymptotic behavior of $k_i(\xi,r,t)$ for $\xi \to \infty$.

Thus, subtracting and adding the asymptotic value of $k_i(\xi,r,t)$ from and to the integrand in the above equation, we have

$$K_i(r,t) = \frac{2}{\pi}\int_0^\infty \{k_i(\xi,r,t) - A_i^*(\xi,r,t)\}\,\mathrm{d}\xi + \frac{2}{\pi}\int_0^\infty A_i^*(\xi,r,t)\mathrm{d}\xi, \tag{5.7}$$

where

$$A_i^*(\xi,r,t) = \frac{\mathrm{e}^{-\xi(2-r-t)}}{\sqrt{rt}}\{\alpha_i(r,t)\xi^2 + \beta_i(r,t)\xi + \gamma_i(r,t)\}, \tag{5.8}$$

and

$$\int_0^\infty A_i^*(\xi,r,t)\,\mathrm{d}\xi = \frac{1}{\sqrt{rt}}\left\{\frac{2\alpha_i}{(2-r-t)^3} + \frac{\beta_i}{(2-r-t)^2} + \frac{\gamma_i}{(2-r-t)}\right\}. \tag{5.9}$$

The form of α_i, β_i, and $\gamma_i (i = 1,\ldots,4)$ comes from the leading terms in the asymptotic expansion of $k_i(\xi,r,t)$ and these are listed in Appendix C. Three terms are retained in the asymptotic forms of the Bessel functions which are used to obtain these values. It is quite onerous work to produce these values. Mathematica is used for this job.

We list in Table 1, the results of subtracting none, one, two, or three of the leading terms in the asymptotic form of the integral. Here we only consider the semi-infinite integral in $K_1(r,t)$. The values of r, and t shown here are those of the nearest values to 1 used in the evaluation of the integral.

If d approaches to zero, from the asymptotic expansions of $K_i(r,t)$ in Appendix C, (4.17) and (4.18) reduce to the following pair of decoupled singular integral equations.

$$\frac{1}{\pi}\int_{-1}^{1}\{g(\tau) - h(\tau)\}\left\{\frac{1}{s-\tau} + \frac{\tau - s}{(2-\tau-s)^2} + \frac{4(1-s)(1-\tau)}{(2-\tau-s)^3}\right\}\mathrm{d}\tau = S, \tag{5.10}$$

$$-\frac{(1-\nu)}{\pi}\int_{-1}^{1}\{g(\tau) + h(\tau)\}\left\{\frac{1}{s-\tau} + \frac{1}{2-\tau-s}\right\}\mathrm{d}\tau = -S. \tag{5.11}$$

Table 1
Sample behavior of semi-infinite integral evaluation when none, one, two, or three of the leading terms in the asymptotic form of integrand in $K_1(r,t)$ are subtracted. $r = 0.9999853$, and $t = 0.9959520$ are the values to produce these data

	Zero-term	One-term	Two-term	Three-term
Asymptotic part	0	1.7739	−122.4279	−120.9037
Gaussian quadrature	0.3323	2.1062	−121.5591	−120.0116

Table 2
Stress intensity factor ($\nu = 0.3$)

d	$\kappa_2/\sqrt{d}$	$\kappa_3/\sqrt{d}$	Keer et al.
0.001	1.1225	−1.0017	1.1224
0.05	1.1367	−1.0569	1.1513
0.1	1.1619	−1.1053	1.1807
0.2	1.2452	−1.2133	1.2608
0.3	1.3806	−1.3604	1.3904
0.4	1.591	−1.577	1.597
0.5	1.924	−1.916	1.932
0.6	2.487	−2.482	2.502
0.7	3.554	−3.551	3.598
0.8	6.063	−6.062	6.201
0.9	15.772	−15.771	16.46

Where $\kappa_2 = k_2/S\cos\theta$, $\kappa_3 = k_3/S\sin\theta$.

Eqs. (5.10) and (5.11) are exactly the solutions of the edge crack problem of a half space for $x \geqslant 0$, when the edge crack occupying $0 < x < 1$, $-\infty < y < \infty$ on the plane $z = 0$ is subjected to the shearing stress while the plane boundary $x = 0$ is stress free. Each equation gives 1.122 and −0.9996 for SIF.

In Table 2 stress intensity factor is listed for $\nu = 0.3$. We list only for $\nu = 0.3$ here, because the variation of SIF with respect to ν is very small. We compare SIF with that by Keer et al. (1977) for the cylinder subjected to tension. From these values we notice that the K values when the cylinder is subjected shear stresses do not differ very much from the values for the uniform tension. Also we notice that the results for $d \to 0$ are in excellent agreement with the analytically obtained limiting value of 1.122.

Figs. 2 and 3 show the crack opening displacements defined by

$$d_1(r) = -\int_a^r G(t)(1-t)^{1/2}(t-a)^{-1/2}t^{-1}\mathrm{d}t + \frac{1}{r^2}\int_a^r H(t)(1-t)^{1/2}(t-a)^{-1/2}t\,\mathrm{d}t = \frac{u(r,\theta,0)}{\cos\theta}\,\frac{\mu}{(1-\nu)S},$$

$$d_2(r) = \int_a^r G(t)(1-t)^{1/2}(t-a)^{-1/2}t^{-1}\mathrm{d}t + \frac{1}{r^2}\int_a^r H(t)(1-t)^{1/2}(t-a)^{-1/2}t\,\mathrm{d}t = \frac{v(r,\theta,0)}{\sin\theta}\,\frac{\mu}{(1-\nu)S}.$$

for the crack depths $d = 1 - a$ of $0.9, \ldots, 0.1$. The maximal displacements $d(1)$ are given in Table 3.

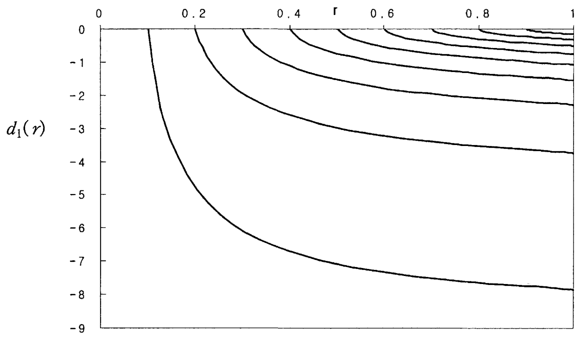

Fig. 2. Crack opening displacement ($\nu = 0.3$).

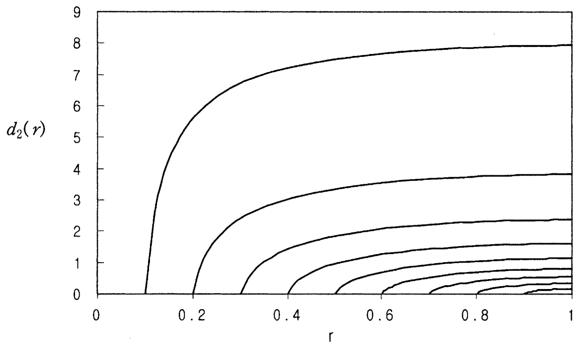

Fig. 3. Crack opening displacement ($\nu = 0.3$).

Table 3
Maximal crack opening displacement for different crack depth ($\nu = 0.3$)

d	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$d_1(r)$	−0.150	−0.316	−0.512	−0.756	−1.080	−1.547	−2.297	−3.744	−7.847
$d_2(r)$	0.157	0.337	0.547	0.804	1.140	1.616	2.374	3.828	7.937

6. Conclusion

In this paper, the problem of determining the distribution of stress in a circumferentially edge cracked circular cylinder which is subjected to uniform shearing stress is considered. This problem does not appear to be investigated previously. As a consequence we were unable to compare the numerical results with any published accounts.

However, comparing present results with the values for uniform tension, we see that the two values are very close. Notifying that the stress intensity factors for the semi-infinite plate with an edge crack are identical for uniform tension and shear stress, we believe that the present results are accurate.

The present analysis can be applied to the stress analysis of cranks, axles, and even pillars or supporters of architectural structures when there are frequent seismic activities.

Appendix A

$$\int_0^\infty \frac{\eta J_1(\eta t)J_1(\eta)}{\xi^2+\eta^2}\,\mathrm{d}\eta = I_1(\xi t)K_1(\xi), \quad t<1,$$

$$\int_0^\infty \frac{J_1(\eta t)J_2(\eta)}{\xi^2+\eta^2}\,\mathrm{d}\eta = -\xi^{-1}I_1(\xi t)K_2(\xi), \quad t<1,$$

$$\int_0^\infty \frac{\eta^2 J_1(\eta t)J_0(\eta)}{(\xi^2+\eta^2)^2}\,\mathrm{d}\eta = \frac{1}{2\xi}\{\xi t I_0(\xi t)K_0(\xi) - \xi I_1(\xi t)K_1(\xi)\}, \quad t<1,$$

$$\int_0^\infty \frac{\eta J_1(\eta t)J_1(\eta)}{(\xi^2+\eta^2)^2}\,\mathrm{d}\eta = -\frac{1}{2\xi^2}[\xi t I_2(\xi t)K_1(\xi) - \xi I_1(\xi t)K_0(\xi)], \quad t<1,$$

$$\int_0^\infty \frac{\eta^2 J_1(\eta t) J_2(\eta)}{(\xi^2+\eta^2)^2}\mathrm{d}\eta = -\frac{1}{2\xi}[\xi t I_2(\xi t) K_2(\xi) - I_1(\xi t)\xi K_1(\xi)], \quad t<1.$$

Appendix B

$$K_1(r,t) = \frac{2}{\pi}\int_0^\infty \xi\{I_1(\xi t)p_1(r,\xi) + \xi t I_2(\xi t)p_2(r,\xi)\}\mathrm{d}\xi,$$

$$K_2(r,t) = \frac{2}{\pi}\int_0^\infty \xi\{I_1(\xi t)p_3(r,\xi) - \xi t I_2(\xi t)p_2(r,\xi)\}\mathrm{d}\xi,$$

$$K_3(r,t) = \frac{2}{\pi}\int_0^\infty \xi\{I_1(\xi t)p_4(r,\xi) + \xi t I_2(\xi t)p_5(r,\xi)\}\mathrm{d}\xi,$$

$$K_4(r,t) = \frac{2}{\pi}\int_0^\infty \xi\{I_1(\xi t)p_6(r,\xi) - \xi t I_2(\xi t)p_5(r,\xi)\}\mathrm{d}\xi,$$

where

$$p_i(r,\xi) = \frac{I_1(\xi r)}{\xi r}a_i(\xi,r) + I_2(\xi r)b_i(\xi), \quad (i=1,\dots,6)$$

with

$$a_1(\xi,r) = -\frac{K_1}{I_1}(1+v+\xi^2 r^2) + \frac{1}{I_1^2} + \xi I_2(I_1\xi - 2I_2)(\xi^2 r^2 - \xi I_2/I_1)/\Delta,$$

$$a_2(\xi,r) = -\frac{K_1}{I_1} - I_1(I_1\xi - 2I_2)(\xi^2 r^2 - \xi I_2/I_1)/\Delta,$$

$$a_3(\xi,r) = \frac{K_1}{I_1}(3-v+\xi^2 r^2) + (-4\xi^2 I_1 I_2 + I_2^2 2(3-v)\xi + I_1^2\xi^3 + \xi^2 r^2\{-I_1 I_2(\xi^2+4-4v) + 2I_2^2\xi\})/\Delta$$

and a_4, a_5 and a_6 are obtained from a_1, a_2 and a_3 respectively by deleting $\xi^2 r^2$.

$$b_1(\xi) = -\frac{K_1}{I_1} + (-I_1^2\xi^3 + I_1 I_2 3\xi^2 - 2\xi I_2^2)/\Delta,$$

$$b_2(\xi) = -\frac{K_1}{I_1} + (I_1^2\xi + I_1 I_2(\xi^2 - 2v) - 2\xi I_2^2)/\Delta,$$

$$b_3(\xi) = \frac{K_1}{I_1} + (I_1^2\xi(4-4v+\xi^2) + I_1 I_2(4v-4-3\xi^2) + 2\xi I_2^2)/\Delta,$$

$$b_4(\xi) = -\frac{K_1}{I_1}v + \xi I_2(I_1\xi - 2vI_2)/\Delta,$$

$$b_5(\xi) = -I_1(I_1\xi - 2vI_2)/\Delta,$$

$$b_6(\xi) = \frac{K_1}{I_1}(2-v) + (I_1^2\xi 4(v-1) + I_1 I_2(4-4v-\xi^2) + 2\xi(2-v)I_2^2)/\Delta$$

and $\Delta = \Delta(\xi)$.

Appendix C

$$A_1^*(\xi,r,t)=\frac{1}{\sqrt{rt}}e^{-\xi(2-r-t)}\left[(1-r)(1-t)\xi^2+\frac{1}{8}\left\{-\frac{3}{t}(1-r)(1-t^2)+4t(1-r)-(1-t)\left(\frac{7}{r}+12-15r\right)\right\}\xi\right.$$
$$+\frac{1}{64}\left(147+\frac{21}{rt}\right)+\frac{57}{128}\left(\frac{1}{r^2}+\frac{1}{t}-\frac{t}{r^2}\right)+\frac{1}{128}\left(\frac{97}{r}-\frac{15}{t^2}+\frac{15r}{t^2}-\frac{75r}{t}-\frac{195t}{r}\right)$$
$$\left.-\frac{11r}{8}-\frac{19t}{4}-v(r-t)+\frac{141rt}{32}\right]\frac{1}{D(\xi)},$$

where

$$D(\xi)=\left\{1-\left(\frac{11}{2}-2v\right)\frac{1}{\xi}-\left(9v-\frac{2025}{128}\right)\frac{1}{\xi^2}-\left(\frac{23787}{1024}-\frac{63v}{4}\right)\frac{1}{\xi^3}\right\},$$

$$A_2^*(\xi,r,t)=\frac{1}{\sqrt{rt}}e^{-\xi(2-r-t)}\left[-(1-r)(1-t)\xi^2-\frac{1}{8}\left\{-\frac{3}{t}(1-r)(1-t^2)+4t(1-r)-(1-t)\left(\frac{7}{r}+12-15r\right)\right\}\xi\right.$$
$$-\frac{1}{64}\left(403+\frac{21}{rt}\right)-\frac{57}{128}\left(\frac{1}{r^2}+\frac{1}{t}-\frac{t}{r^2}\right)-\frac{1}{128}\left(\frac{97}{r}-\frac{15}{t^2}+\frac{15r}{t^2}-\frac{75r}{t}-\frac{195t}{r}\right)$$
$$\left.+\frac{43r}{8}+\frac{19t}{4}+v(4-3r-t)-\frac{141rt}{32}\right]\frac{1}{D(\xi)},$$

$$A_3^*(\xi,r,t)=\frac{1}{\sqrt{rt}}e^{-\xi(2-r-t)}\left[\frac{(1-r)(1-t)}{r}\xi+\frac{1}{8}\left(-\frac{3}{rt}-\frac{3}{r^2}+\frac{15t}{r}+12v+3-6t-16vt-\frac{8}{r}+\frac{3}{t}+\frac{3t}{r^2}\right)\right]$$
$$\times\frac{1}{D(\xi)},$$

$$A_4^*(\xi,r,t)=\frac{1}{\sqrt{rt}}e^{-\xi(2-r-t)}\left[-\frac{(1-r)(1-t)}{r}\xi+\frac{1}{8}\left(\frac{3}{rt}+\frac{3}{r^2}-\frac{15t}{r}-20v+5+6t+16vt+\frac{8}{r}-\frac{3}{t}-\frac{3t}{r^2}\right)\right]$$
$$\times\frac{1}{D(\xi)}.$$

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